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- 201. Use the cosine rule.
- 202. Find the gradient of the line.
- 203. The possibility space has $2^4 = 16$ equally likely outcomes. List them and use $p = \frac{\text{successful}}{\text{total}}$.
- 204. Factorise.
- 205. Are there any $x \in \mathbb{R}$ which cannot be reciprocated?
- 206. Set up NII and solve for m.
- 207. Leave the denominator as it is, and factorise the numerator as a difference of two squares.
- 208. Consider the LHS S(n-1) + n as an expression, and simplify by factorising. You want $\frac{1}{2}n(n+1)$.
- 209. "Mutually exclusive" means the events cannot both happen. To show that (3) is impossible, use the inclusion-exclusion formula

$$\mathbb{P}(A\cup B)=\mathbb{P}(A)+\mathbb{P}(B)-\mathbb{P}(A\cap B)$$

- 210. Consider a full journey around the perimeter (total angle 2π), divided up into 10 turns at the vertices.
- 211. Scientific calculators can evaluate these quantities directly. If, when you try to enter data, there is no frequency column on your calculator, there is probably a setting to toggle frequency on/off.
- 212. (a) Use the standard integration formula

$$\int x^n \, dx = \frac{1}{n+1}x^{n+1} + d$$

- (b) Substitute (-1, 5) and solve for d.
- 213. The general formula for the *n*th term of an AP is $u_n = a + (n-1)d$. Substitute this and a similar formula for u_{n-1} into the expression for w_n . Then simplify to the same arithmetic form, with first term 2a d and common difference 2d.
- 214. Consider that reflection in y = x sends point (a, b) to point (b, a).
- $215.\ {\rm Construct}$ a counterexample to the claim.
- 216. The answer is independent of a.
- 217. In an identity, the coefficients of each power of x must be the same on both sides of the equation: so, one identity can generate multiple equations. In this case, consider the x term.

- 218. (a) Consider division by zero.
 - (b) You'll get an expression in terms of y. At the end, rewrite this with x as the input.
- 219. Substitute the linear equation into the equation of of the circle, and solve the resulting quadratic.
- 220. Find an expression for the shaded area in terms of the radius a of the quarter circle. Set this to $\frac{1}{2}$.
- 221. Place 1 without loss of generality (the pentagon is symmetrical, so it makes no difference where). Then consider 2.
- 222. Calculate the resultant force using NII. Compare it to the magnitudes of the individual forces.
- 223. Solve simultaneously, then use Pythagoras.
- 224. Begin with the form $y = k(x-p)^2 + q$.
- 225. Determine the probability that the first r-1 rolls are all non-sixes and the probability that the rth roll is a six. Multiply these together and simplify.
- 226. (a) An asymptote is a line approached by a curve, such that the distance between asymptote and curve tends to zero. Consider $x \to -\infty$.
 - (b) Set up $2^x 4^x = 0$ and solve by factorising.
 - (c) Consider the behaviour as $x \to \infty$. Put this together with the results from (a) and (b).
- 227. Use the formula $p = \frac{\text{successful}}{\text{total}}$.
- 228. The fact that the sequence is an AP isn't relevant. Consider the relationship between the mean, sum and number of terms of a sequence or data set. There is no need to find (or indeed possibility of finding) individual terms or common differences.
- 229. Remember that a x = -(x a).
- 230. Use integration.
- 231. (a) Give the name of a quadrilateral which is both a kite and a parallelogram.
 - (b) Give the name of a prime number which is a multiple of 11.
- $232. \ {\rm Find}$ the perpendicular bisector of the points.
- 233. Rearrange to LHS = 0 and factorise the LHS. Then use the factor theorem.
- 234. The gradients of perpendicular lines are negative reciprocals. Equivalently, their product is −1.

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235. Find the area of an equilateral triangle in terms of edge length, using ¹/₂ab sin C. Multiply by 8.
236. It is possible to determine the value.
237. Use a symmetry argument on a unit circle or a graph. The values simplify nicely.

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- 238. The roots of this expression are the x values that make it zero. The second factor offers no real roots.
- 239. Use the horizontal information to find the length of time it takes for him to cross the ravine. Then look at the vertical motion.
- 240. It's the second line to the third line.
- 241. f''(x) is the derivative of f'(x). Split the fraction up and differentiate.
- 242. Combine the forces according to Pythagoras.
- 243. Set $p = \frac{a}{b}, q = \frac{c}{d}$, where $a, b, c, d \in \mathbb{Z}$. Multiply these together and simplify. Show that the result is the quotient (fraction) of two integers.
- 244. Consider the gradients of the lines. If two straight lines have different gradients, they must intersect.
- 245. Expand and simplify the LHS, or rearrange and take the square root.
- 246. The (signed) area between a velocity-time graph and the t axis is equal to the displacement.
- 247. Double the first inequality. Then explain why, if this new inequality holds, the second inequality must too.
- 248. (a) Sketch the scenario. Consider the gradient of the radius joining the origin to the new point.
 - (b) The squared distance from the point (2k, k) to the origin must be 20.
- 249. 4.3% "compounded every quarter" means that the scale factor every quarter-year is 1.01075.
- 250. Calculate a definite integral.
- 251. How often do factors of 7 appear?
- 252. Consider the hexagon as six equilateral triangles, each of side length 1. Find the angles and then side lengths of the four unshaded triangles.
- 253. Write x in terms of y, and substitute.

- 254. Imagine picking the letters one at a time, without replacement: both need to be "not E".
- 255. Consider the angles involved when three circles of equal radius are placed tangent to one another.
- 256. (a) Use the differentiation rule for polynomials.
 - (b) Sub x = 2 into $\frac{dy}{dx}$.
 - (c) Sub x = 2 into the equation of the parabola.
 - (d) Use $y y_1 = m(x x_1)$.
- 257. Draw a clear sketch, placing A, B, C at arbitrary points. Use the fact that, for any points P and Qwith position vectors \mathbf{p} and \mathbf{q} , the vector from Pto Q is given by $\overrightarrow{PQ} = \mathbf{c} - \mathbf{b}$.
- 258. When dividing one factorial by another, many of the factors cancel.
- 259. Consider the factor theorem. Does the implication go both ways?
- 260. Start with $(x 1)^2$. Multiply this out, then deal with the x and constant terms.
- 261. The locus is the perpendicular bisector of (4,0) and (0,4). These are symmetrical in x and y.
- 262. (a) Consider similar triangles.
 - (b) The identity is $\sin^2 \theta + \cos^2 \theta \equiv 1$.
- 263. For a fraction to be zero, it is necessary that the numerator be zero.
- 264. Here, the word "reaction" doesn't mean exactly what Newton originally meant in his third law.
- 265. A curve has a vertical asymptote where division by zero occurs. Consider the value of $x^2 + x + k$ at x = 4.
- 266. Set up the usual calculation for percentage error, setting your calculator to radian mode to evaluate the true value of the cosine function.
- 267. Calculate u_1 , then u_2 , then u_3 .
- 268. An algebraic fraction, which is in its lowest terms, is zero if and only if its numerator is zero. Hence, explain, with reference to the denominator, why this particular fraction can be *guaranteed* to be in its lowest terms.
- 269. Using NII, the acceleration is a negative quadratic in t. Either factorise, or complete the square, or use calculus to find its vertex.

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- 270. First, sketch $y = x^2 + 1$, then switch x and y, thereby reflecting in the line y = x.
- 271. This is a one-dimensional stretch. Think whether it is an input or an output transformation, and find the length scale factor in that direction. Since the other dimension is unaffected, the area scale factor is the same as the length scale factor.
- 272. A heptagon can be divided up into five triangles.
- 273. Simplify the individual surds first.
- 274. Use 3D Pythagoras.
- 275. The range is the set of outputs attainable from a given domain. If in doubt, sketch a graph of $y = x^2$. Consider the domain as a subset of the x axis, and the range the resulting subset of the yaxis.
- 276. First, calculate the gradient and midpoint of the line joining the given points. Then use the formula $y y_1 = m(x x_1)$.
- 277. Consider the number of steps from 1 to n.
- 278. Put the algebraic fraction in its lowest terms, then set the numerator to zero.
- 279. "Monic" means having a leading coefficient of 1.
- 280. The possibility space is a 6×6 table of 36 equally likely outcomes. Having drawn such a table, locate the successful outcomes and use $p = \frac{\text{successful}}{\text{total}}$.
- 281. Set up vertical and horizontal equations of motion. Divide one by the other.
- 282. An ellipse is closed, so evaluating the LHS will show if a point is inside, on, or outside it.
- 283. An AP is an arithmetic progression, for which the difference between terms is constant. Consider the mean of the side lengths.
- 284. Find $\sum x$ using the formula $\bar{x} = \frac{\sum x}{n}$. Add the new mark to $\sum x$ and recalculate.
- 285. Set the denominator to zero and solve.
- 286. Consider the implication from right to left, which doesn't hold for all three of these.
- 287. The region is circular.

- 288. Differentiate the first curve. For the second curve, multiply out before differentiating. Then equate the two derivatives and solve a quadratic.
- 289. Assume that the domain is \mathbb{R} . Over this domain, the range of sin x is [-1, 1]. Consider the effect on this range of subsequent operations.
- 290. (a) Use the negative reciprocal gradient for i. and write down the answer for ii.
 - (b) Solve the equations in (a) simultaneously.
 - (c) Use Pythagoras to find squared distances.
 - (d) A cyclic quadrilateral is one whose vertices are on the circumference of the same circle.
- 291. Provide a counterexample: two distinct functions with the same derivative.
- 292. "Concurrent", describing three or more graphs, means having a mutual point of intersection.
- 293. Just sketch a few triangles to see what's going on, then explain. Consider the maximum number of intersections that a line can have with a triangle. You might want to consider that every triangle is convex, i.e. each interior angle is acute/obtuse.
- 294. The result required contains no t, so rearrange the simpler of the two equation to make t the subject, substitute and simplify.
- 295. Take out a factor of x first, to deal with the root x = 0. You then need the remaining quadratic to have exactly one root.
- 296. Consider the congruency of the triangles.
- 297. If in doubt with integration, consider the process as anti-differentiation. Ask the question "What would differentiate to give this?"
- 298. Consider extreme cases:
 - (a) one of the sides is tiny,
 - (b) all the sides have the same length.
- 299. Two statements are true; one is false. Remember that the definitions of quadrilaterals overlap, so that e.g. a square is also a rectangle.
- 300. Draw $y = x^2$ first, then consider transformations.

- End of 3rd Hundred —

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